to be 330 Ω/□. Thus, for each model parameter, a scaling rule is derived. These scaling rules are compared with the process data in Table I. The process parameters are determined from test structures of the process control monitor of the Siemens HBT process. The process data range, e.g., for the base contact, includes variations both of process inhomogeneity and base layer properties along a whole wafer. The deviations of the small-signal model parameters from the regression line are due to the arbitrary selected sample. Nevertheless, the scaling parameters compare favorably with the process data, the agreement being better than 10%. The bias condition served as a basis for the derivation of the scaling rules; these rules are not restricted to this bias point, they apply to the normal active region of the transistor. The self-heating effect of the HBT is considered in two aspects. First, the HBT's are operated under constant current density to achieve almost equal conditions for a comparison. Second, the larger the HBT area, the more pronounced the self-heating effect for the device. This feature is captured in the decrease of the current source parameters α_0 and $F_{3\mathrm{dB}}$ with the increase of the area (see Table I).

V. Conclusion

Physical scaling rules for AlGaAs/GaAs power HBT's with 2–16 emitter fingers of 120–960-μm^2 emitter areas have been developed. The parameter extraction method was based on a small-signal T-shaped equivalent circuit. From the small-signal model, scaling rules with scaling parameters has been established based on the physical interpretation of each equivalent circuit parameter. The scaling parameters compare favorably with the measured data from the process control monitor. The scaling rules can provide a basis for layout of power transistors and for the control of critical performance parameters.

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Calibration and Verification of the Pure-Mode Vector Network Analyzer

David E. Bockelman and William R. Eisenstadt

***Abstract*—In this paper, the calibration of a pure-mode vector network analyzer (PMVNA) is presented in detail. The analyzer is intended for the measurement of mixed-mode scattering parameters (s-parameters) of differential circuits, but is also suitable for measurement of general microwave networks with up to four ports. The theory of calibration of the analyzer is developed in terms of a general n-port analyzer, including the correction of port-to-port crosstalk. The type of the standards used in calibration is examined, and the minimum number of standards are summarized for various levels of crosstalk correction. A new standard—called a generalized through, desirable for all multiport network analyzer calibrations—is introduced. A calibration is performed from 0.25 to 25.25 GHz based on standards with coaxial connectors, and verification standards are measured. The measured data is compared with National Institute of Standards and Technology (NIST) traceable measurements, and errors are found to generally less than ±1 dB in transmission. In many cases, the error is less than the uncertainty of the NIST traceable measurements.**

***Index Terms*—Calibration, measurement, measurement standards, networks.**

I. Introduction

Differential circuits are becoming increasingly important in radio frequency (RF) and microwave applications, particularly in integrated circuits (IC's). The differential circuit topology is being widely adopted in RF IC's due to its crosstalk immunity and increased dynamic range over ground-referenced circuits. This increase in differential applications at RF has lead to the development of scattering parameter (s-parameter) based characterization of these circuits, known as mixed-mode s-parameters [1].

Accurate measurements are ultimately required for any RF differential application, but the measurement of differential circuits has been a significant problem. Recently, a new specialized vector network analyzer (VNA) system has been developed for the measurement of differential circuits [2]. This new analyzer, called a pure-mode VNA (PMVNA), stimulates and measures the device under test (DUT) with the two fundamental modes of differential circuit operation: the differential mode and the common mode. Special considerations required for the accurate calibration of the PMVNA and the results of a practical implementation of the PMVNA calibration are presented in this paper. This paper is organized as follows. In Section II, the error model of the PMVNA is presented and the basic calibration equation is given. The removal of switching effects is also covered in this section. Section III provides details of the solution of the calibration problem. Section IV presents measured results and verification through comparison to National Institute of Standards and Technology (NIST) traceable data, and conclusions are given in Section V.

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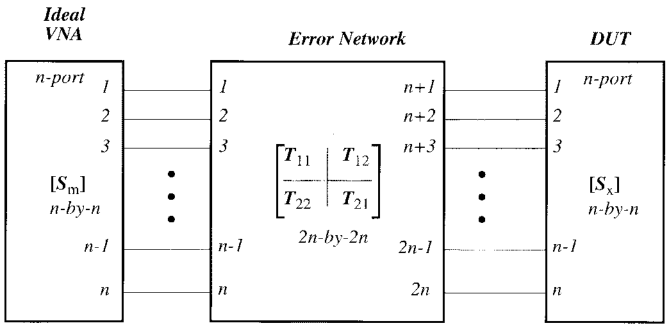

Fig. 1. Error model for the general n-port VNA.

II. PMVNA Error Model

Before calibration of the PMVNA can be attempted, an error model is needed. A generalized error model for an n-port network analyzer, including all port-to-port leakage errors, has been introduced by Speciale [3]. The error model, shown in Fig. 1, employs an error network with $2n$ ports, and the error network is an equivalent representation of all linear repeatable errors. All repeatable errors in the PMVNA can be represented with the generalized error model, with n equal to four. In the case of the PMVNA, the error network is a mixed-mode representation, as defined in [1]. A similarity transform relates the error matrices of the two-port PMVNA and the standard four-port VNA [2], thus the applicability of the error model of Fig. 1 is assured. Due to this transformation, the calibration theory of the PMVNA parallels that of a standard four-port VNA.

Each signal path in the error network is an error term which must be found during calibration. The error model of Fig. 1 includes all possible error terms, including all port-to-port leakage paths. For an n-port VNA, there are $4n(n-1)$ leakage terms out of a total of $(2n)^2$ error terms. It is important to address the PMVNA calibration problem in the most general terms, but there are some simplifications of the error model that are also of interest where some of the leakage paths are neglected. One such simplification of the error model neglects all leakage between ports, which will be called the no-leakage model, leaving a total of n^2 error terms.

In general, VNA calibrations require the application of multiple calibration standards. With the use of the measured and assumed known actual s-parameters of the standards, all error terms are to be found. An algebraic solution approach to the general n-port network analyzer calibration problem has also been introduced by Speciale [3]. When the error network of Fig. 1 is expressed in terms of chaining s-parameters (called t-parameters), the relationship between measured s-parameters (with errors) and actual (errorless) s-parameters can be expressed by

$$T_{11}S_a + T_{12} - S_m T_{21} S_a - S_m T_{22} = 0 \tag{1}$$

where S_a is the actual s-parameter matrix, S_m is the measured s-parameter matrix, and T_{ij} are the four equal partitions of the error network $\boldsymbol{T}$. In terms of an n-port VNA, the error network is represented by a $2n$-by-$2n$ unknown network, and each s-parameter matrix is an n-by-n matrix. The matrix equation (1) can be applied to all standards, and expanded, with the resulting scalar equations linear in the elements of $\boldsymbol{T}$. The set of all scalar equations can then be rewritten as

$$\boldsymbol{D} \cdot \bar{t} = \bar{0} \tag{2}$$

where $\bar{t}$ is a column vector comprised of the elements of $\boldsymbol{T}$ [4], [5]. Given that m different calibration standards are applied, the coefficient matrix $\boldsymbol{D}$ has dimensions (mn^2)-by-$(2n)^2$, and $\bar{t}$ has dimensions $(2n)^2$-by-1.

The development of the calibration equation (1) is predicated on the assumption that the error model remains static throughout the calibration process and through any subsequent measurements. The PMVNA (and most other automatic VNA's) use RF switches to set the stimulus mode.[1] However, changing the switch positions violates the primary assumption of the error model. These switching errors must be effectively removed before the error model can be applied to a calibration. In addition, imperfect pure-mode generation can lead to violation of the static assumption. As shown in [2], the PMVNA generates the differential and common-mode stimuli from a 0°/180° hybrid power splitter. It has been shown that any imbalances in the splitter, together with any phase and magnitude imbalance in the PMNVA, will generate a spurious mode simultaneously with the desired mode [6]. These imbalances, which change as the RF switches change, can cause significant inconsistencies in the raw mixed-mode s-parameters. For accurate calculation of raw mixed-mode s-parameters, any imperfections and changes in the stimulus must also be removed.

The removal of the switching effects and the stimulus imbalance can be achieved through the application of all eight samplers in the PMVNA. This approach is an extension of two-port VNA techniques [7]. By using measurements at all samplers for each switch position, the effects of the switch and mode imbalance can be removed from the measured s-parameters.

III. Solution of the Calibration Problem

The construction of the calibration equation (2) and its subsequent solution are the heart of calibration process. The conditions under which a solution to (2) can be found will first be presented in terms of a general n-port problem. These conclusions will then be applied to the calibration of the PMVNA.

For purposes of this paper, a solution is valid for calibration only if it is unique within one arbitrary scalar. That is, if $\bar{t}_1$ is a valid solution vector of (2), then the only other solution vectors that exist are $\alpha\bar{t}_1$, where α is any complex scalar. In other words, the null space of $\boldsymbol{D}$ must be of dimension one. For ease, this type of solution will be called an ordinary solution. For an n-port calibration, a single standard will be considered to always have n-ports, regardless of actual construction of the physical standard. For example, an n-port match standard may be constructed of a group of n independent one-port match loads, but for purposes of discussion, the group will be considered as a single standard. Because of the special case of no leakage, calibration standards will be considered to be either a reflection or transmission standard. A reflection standard is defined as a group of n one-port reflection standards (such as the n-port match example above); a transmission standard is defined to have transmission between at least two ports. For the full error model, both types of standards are treated the same, but in the case of the no-leakage model, the reflection standards generate fewer sets of measurement data (hence, fewer equations) than the transmission standards.

The determination of the number of required standards for calibration is based on consideration of the rank of the coefficient matrix $\boldsymbol{D}$. For an ordinary solution to (2), matrix $\boldsymbol{D}$ must have a rank of exactly $(2n)^2 - 1$. For the full error model, each standard generates n^2 equations. It can be shown [8] that $\boldsymbol{D}$ will have rank of exactly $(2n)^2 - 1$ only with five or more n-port standards. This means that $5n^2$ equations are generated for the solution of $4n^2$ error terms, so the system of equations (2) is overdetermined. This is consistent with updated results from Speciale [9]. Conclusions about the no-leakage

[1] *HP 8510C Network Analyzer: On-Site Service Manual*, Hewlett-Packard, Santa Rosa, CA, Aug. 1991.

TABLE I
CALIBRATION SUMMARY

Error Model Type	Number of Unknowns	Eqns from Stds: Transmission	Eqns from Stds: Reflect	Minimum Number of Stds for Ordinary Solution
Full-leakage	$4n^2$	n^2	n^2	5
No-leakage	$4n$	n^2	n	depends on n

model are not as general, since the minimum number of required standards depends on the number of ports n. For example, if n is 4 (as for the PMVNA), at least two transmission standards are required for rank of $4n-1=15$. In contrast, if n equals 2, at least three standards are required (e.g., line–reflect–match (LRM) [4]). All conclusions are summarized in Table I.

Of course, the rank of the coefficient matrix $\boldsymbol{D}$ is effected by the type of each standard used in calibration. For error models like Fig. 1, it has been found [8] that at least one of the standards must be constructed so that at least $n-1$ nonzero transfer functions exist between its ports. In other words, such a standard interconnects all of the VNA's measurement ports simultaneously. Hence, this standard will be called a generalized through standard. It is important to note that the generalized through does not necessarily provide low-loss interconnection between the ports. Without a generalized through standard as one of the minimal set of standards, the rank of the coefficient matrix will not be sufficient for an ordinary solution to the calibration equation. As its name implies, the generalized through is a generalization of the through standard of two-port VNA calibrations. For a two-port VNA, the through provides transfer between all ports. However, with more than two ports, the generalized through standard is less familiar. It can be shown that one or more two-port throughs used as a single n-port standard are not sufficient for an ordinary solution to (2). Furthermore, use of multiple pairs of throughs, each connecting different ports, will not provide sufficient rank in $\boldsymbol{D}$ for an ordinary solution.

The PMVNA calibration has been implemented with the application of the above conclusions about general VNA calibrations. With n being four, the PMVNA calibration standards can be defined in physical terms. For this paper, calibration standards with 3.5-mm coaxial connectors are used. Five standards are combined to create the calibration kit:

1) four-port match (four 50-Ω loads);
2) four-port short (four offset shorts);
3) four-port open (four offset opens);
4) pair of zero-length through lines (by connecting test cables of ports 1–3 and 2–4);
5) four-port resistive star network as the generalized through.

The star network is constructed from two resistive power dividers[2] connected as shown in Fig. 2. For this paper, the assumed actual s-parameters of the star network have been found through "round-robin" two-port VNA measurements.

After measurement of the calibration standards, the solution to (2) is found using singular value decomposition [11]. After the error matrix $\boldsymbol{T}$ is found, any subsequent device measurements can be corrected through the application of (1). For the PMVNA, the s-parameters are expressed in terms of mixed-mode s-parameters, but

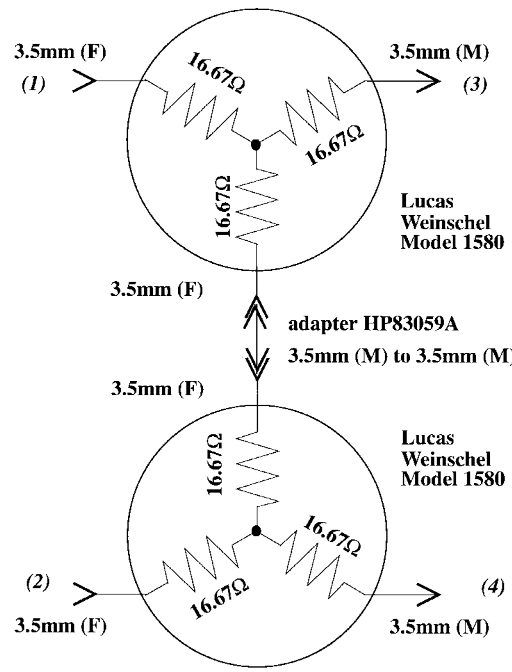

Fig. 2. Schematic of coaxial star-network standard.

the corrected s-parameters can be transformed into standard four-port s-parameters if desired.

IV. RESULTS AND VERIFICATION

To provide a verification of the accuracy of the PMVNA calibration, it is required to measure some standard other than those used in calibration. The verification standards used here are provided by a Hewlett-Packard 85 057B verification kit.[3] This kit contains four two-port standards, each accompanied with NIST traceable s-parameter measurements (these measurements have associated uncertainties, also provided). For verification of the PMVNA, various combinations of two verification standards are measured, and the corrected measurements are compared to the provided s-parameters. While combinations of two-port devices does not represent a general differential device that the PMVNA is designed to measure, the 85 057B kit provides a readily available means of accuracy verification. The verification standards have 2.4-mm coaxial connectors, in contrast to the 3.5-mm connectors of the PMVNA, so adapters have been used on all verification standards. These adapters have been characterized and deembedded from all measurements. The verification measurements are made by the PMVNA, which has been calibrated using all five

[2] *Product Literature: Power Divider Model 1580,* Lucas Weinschel Corporation, Gaithersburg, MA.

[3] *HP 85057B 2.4 mm Verification Kit Product Manual,* Hewlett-Packard, Santa Rosa, CA, Feb. 1991.

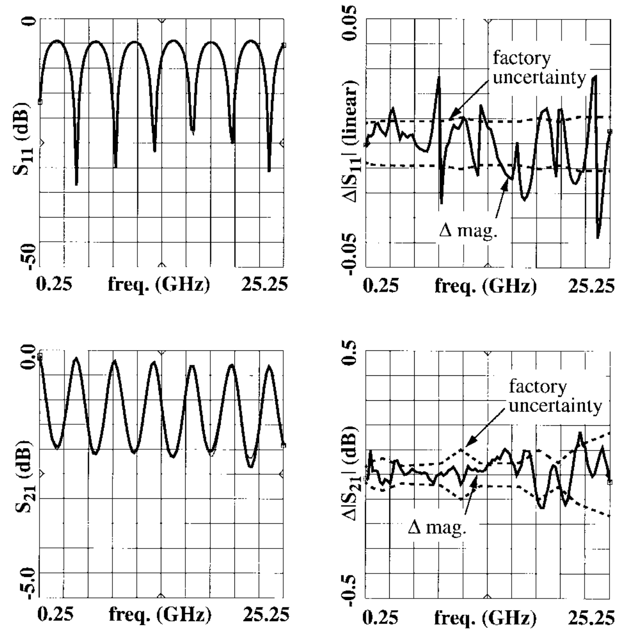

Fig. 3. (a) Measured s-parameters with adapters deembedded (bold) and verification s-parameters of the 25-Ω air–dielectric transmission line, connected between ports two and four while the 50-Ω air–dielectric transmission line is connected between ports one and three. (b) Differences between measured s-parameters and verification s-parameters of (a) (solid) and factory uncertainty of verification s-parameters (dashed). Errors in S_{11} expressed as the difference of the linear magnitudes of the respective data. Errors in S_{21} are expressed as the difference in decibels of magnitudes in decibels.

standards described earlier, with perfect isolation between the port assumed. All measurements (calibration and verification) are made with 1024 averages.

The first verification standard measurement is the simultaneous measurement of the 50-Ω air–dielectric transmission line and the 25-Ω air–dielectric transmission line. The 50-Ω transmission line is connected between ports one and three, while the 25-Ω transmission line is connected between ports two and four. Due to space limitations, only some of the measured s-parameters of the 25-Ω transmission line are shown in Fig. 3(a), together with the s-parameters provided with the verification kit. The agreement between the two sets of data is quite good, and the errors are shown in Fig. 3(b). The magnitude error of S_{11} is less than about ± 0.04, which is good considering the large variation in the magnitudes of the parameter. S_{21}, which also varies significantly over the measurement, has less than ± 0.2-dB magnitude error, which compares reasonably to the uncertainty, and no more than 5° phase error (not shown) with respect to the s-parameters provided with the verification kit.

The measurements of other traceable verification standards are given in [8], and all compare well.

V. Conclusions

The calibration of the PMVNA has been shown to be accurate in terms independent of the PMVNA. Strictly speaking, the accuracy of the calibration has been established for only the specific verification standards measured. These verification standards are meant to represent some extremes of possible DUT performance, so it is argued that the accuracy of the measurements of any DUT can be reasonably assured. The verification standards as shown do not exercise all of the 16 s-parameters measurable. However, many other combinations of the same verification standards have been made. These measurements have not been shown due to space limitations, but all compare to verification data with the same general level of accuracy. It is argued that these many measurements reasonably verify the accuracy of all 16 s-parameters measured by the PMVNA.

The calibration of the PMVNA has been successfully completed in the theoretical framework of a general VNA calibration. The appropriateness of this approach has been established through theoretical arguments and validated through measured results. The requirements for a solution of a general PMVNA calibration problem have been established, and new approaches to calibration standards have proven accurate. In addition, a new type of calibration standard—the generalized through—has been introduced. The generalized through is required for any general n-port calibration using the minimum number of standards.

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